



Mark Scheme

Summer 2023

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 27 : Decision Mathematics 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - **B** marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
 - ft – follow through
 - the symbol \checkmark will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

| Question | Scheme | Marks | AOs |
|---|--|--------------|------|
| 1 | Middle right | Pivot(s) | |
| | 67 59 46 71 40 48 53 63 45 54 56 | 48 | |
| | 67 59 71 53 63 54 56 <u>48</u> 46 40 45 | 53, 40 | M1 |
| | 67 59 71 63 54 56 <u>53</u> <u>48</u> 46 45 <u>40</u> | 63, 45 | A1 |
| | 67 71 <u>63</u> 59 54 56 <u>53</u> <u>48</u> 46 <u>45</u> <u>40</u> | 71, 54, (46) | A1ft |
| | <u>71</u> 67 <u>63</u> 59 56 <u>54</u> <u>53</u> <u>48</u> 46 <u>45</u> <u>40</u> | (67), 56 | A1 |
| | <u>71</u> 67 <u>63</u> 59 <u>56</u> <u>54</u> <u>53</u> <u>48</u> 46 <u>45</u> <u>40</u> | | |
| | Middle left | | |
| | 67 59 46 71 40 48 53 63 45 54 56 | 48 | |
| | 67 59 71 53 63 54 56 <u>48</u> 46 40 45 | 53, 40 | |
| | 67 59 71 63 54 56 <u>53</u> <u>48</u> 46 45 <u>40</u> | 71, 46 | |
| | <u>71</u> 67 59 63 54 56 <u>53</u> <u>48</u> <u>46</u> 45 <u>40</u> | 63, (45) | |
| | <u>71</u> 67 <u>63</u> 59 54 56 <u>53</u> <u>48</u> <u>46</u> 45 <u>40</u> | (67), 54 | |
| | <u>71</u> 67 <u>63</u> 59 56 <u>54</u> <u>53</u> <u>48</u> <u>46</u> 45 <u>40</u> | 59 | |
| | <u>71</u> 67 <u>63</u> <u>59</u> 56 <u>54</u> <u>53</u> <u>48</u> <u>46</u> 45 <u>40</u> | | |
| | | (4) | |
| (4 marks) | | | |
| Notes for Question 1 | | | |
| <p>M1: Quick Sort, pivot, p, chosen (must be choosing middle right or middle left). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). If choosing only one pivot per iteration, then M1 only. Their original list of 10, 11 or 12 numbers must be a list of 10, 11 or 12 numbers after the first pass</p> <p>A1: First two passes correct (pivots for third pass need not be chosen)</p> <p>A1ft: Third and fourth passes correct (follow through from their second pass and choice of consistent pivots) (pivot(s) for the fifth pass need not be chosen). After their second pass their list must contain either 10, 11 or 12 numbers (so allow one additional/missing number)</p> <p>A1: cso – if choosing middle right pivots then they must include a fifth pass and if choosing middle left then they must include a sixth pass</p> <p>SC: If list is sorted into ascending order, then award a maximum of M1A1A0A0 (so 2 marks) as in the scheme above even if the list is re-ordered after the sort is complete</p> <p>Ascending: 67 59 46 71 40 48 53 63 45 54 56</p> <p>46 40 45 <u>48</u> 67 59 71 53 63 54 56</p> <p><u>40</u> 46 45 <u>48</u> <u>53</u> 67 59 71 63 54 56</p> | | | |

| Question | Scheme | Marks | AOs |
|-------------------|---|----------------------|-----------------------------|
| 2(a) | The dummy from event 3 to event 4 is required as activity F depends only on activity C, but activities G, H and J depend on activities C, B and E | B1 | 2.4 |
| | | (1) | |
| 2(b) | | M1 A1 A1 | 1.1b 1.1b 1.1b |
| | | (3) | |
| 2(c) | Critical activities are A, E, J and K | B1 | 1.1b |
| | | (1) | |
| 2(d) | $\frac{6+10+7+7+8+11+5+9+2+6+9+5}{29} = \frac{85}{29} = 2.931... \text{ so a lower bound of 3 workers}$ | B1ft | 2.2a |
| | | (1) | |
| 2(e) | | M1 A1 A1 A1 | 2.1 1.1b 1.1b 1.1b |
| | | (4) | |
| (10 marks) | | | |

Notes for Question 2

(a)

B1: Correct reasoning for the dummy activity – must mention activities F and C (twice or clearly implied twice), at least one of B/E, and at least one of G/H/J (for example, ‘F relies on C, but G relies on C and E’)

(b)

M1: All top boxes and all bottom boxes completed. Values generally increasing left to right (for top boxes) and values generally decreasing from right to left (for bottom boxes). Condone missing 0s at the source node or the 29 in the bottom box at the sink node for the M mark only. Condone one rogue value in top boxes and one rogue value in bottom boxes. For a rogue in the top boxes if values do not increase in the direction of the arrows, then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value (with a similar definition for bottom boxes but in reverse)

A1: cao - Top boxes (including zero at the source node)

A1: cao - Bottom boxes (including zero at the source node and 29 at the sink node)

(c)

B1: cao (the correct four critical activities A, E, J and K and no others)

(d)

B1ft: Correct deduction of lower bound from a correct calculation for **their** minimum project completion time from **(b)** (so if correct in **(b)**, must be 85/29). The follow through is on their 29 only (so no follow through for incorrectly adding up the duration of all the activities). An answer of 3 with no working scores no marks. All working seen must be correct. As a minimum must either see

$$\frac{6+10+7+7+8+11+5+9+2+6+9+5}{29} \text{ or } \frac{85}{29} \text{ or an awrt } 2.9 \text{ (not from incorrect working) followed}$$

by 3

(e)

M1: At least nine different activities labelled including at least five floats. A scheduling diagram (so a diagram in which no floats are evident) scores M0

A1: The critical activities dealt with correctly and appearing just once (A, E, J and K) and three non-critical activities dealt with correctly (both duration and total float correct)

A1: Any six non-critical activities correct (this mark is not dependent on the previous A mark)

A1: cso – completely correct Gantt chart (all twelve activities appearing exactly once)

For **(e)** the following may be useful in checking their cascade chart provided the float is shown after the corresponding activity:

| Activity | Duration + Float |
|----------|------------------------|
| A | 0 to 6 Critical |
| B | 0 to 10 F: 10 to 14 |
| C | 0 to 7 F: 7 to 9 |
| D | 6 to 13 F: 13 to 22 |
| E | 6 to 14 Critical |

| Activity | Duration + Float |
|----------|-------------------------|
| F | 7 to 18 F: 18 to 20 |
| G | 14 to 19 F: 19 to 22 |
| H | 14 to 23 F: 23 to 24 |
| I | 19 to 21 F: 21 to 24 |
| J | 14 to 20 Critical |

| Activity | Duration + Float |
|----------|-------------------------|
| K | 20 to 29 Critical |
| L | 23 to 28 F: 28 to 29 |

| Question | Scheme | Marks | AOs |
|-------------|--|--|------------------------------|
| 3(a) | Dijkstra's algorithm only finds the shortest path from a given starting node to all other nodes in the network. If the shortest route from A to J via C is required, then this is equivalent to finding the shortest paths from C to A and C to J and so therefore as C is common to both paths then the algorithm should start at C | B1 | 3.5b |
| | | (1) | |
| (b) | | M1 A1 (CDBH) A1 (GEA) A1ft (FJ) | 1.1b 1.1b 1.1b 1.1b |
| | Shortest route from A to J via C is A B D C D G E F J | A1 | 2.2a |
| | Shortest length: $70 + 86 = 156$ (km) | A1ft | 2.2a |
| | | (6) | |
| (c) | Prim's algorithm from C: CD, BD, DG; EG, GH, EF; FJ, AB | M1 A1 A1 | 1.1b 1.1b 1.1b |
| | | (3) | |
| (d) | 155 (km) | B1 | 1.1b |
| | | (1) | |

(11 marks)**Notes for Question 3****(a)**

B1: Recognition that the limitation of the algorithm is that it can only find the shortest path from a given starting node to all others (and not from any node to any other node) and therefore a clear indication is required that C is the starting vertex to of the two required paths that include A and J. As a minimum accept that finding A to J via C is equivalent of finding the shortest (oe) path from C to J and C to A (these two paths **must** be stated this way round)

In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 58 50 in that order (so 50 58 is incorrect)

It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling

(b)

M1: A larger working value being replaced by a smaller working value at any **two** of A, B, E, F, G, J

A1: All values at C, D, B and H correct and the working values in the correct order

A1: All values at G, E and A correct and working values in the correct order. Penalise order of labelling only once per question. Condone an additional working value of 57 after the 39 (but **A0** if the 57 appears before either the 41 or 39 at G)

A1ft: All values in F and J correct on the follow through and the working values in the correct order. To follow through F check that the working values at F follow from the candidate's final values for the nodes that are directly attached to F (which are B, E, G and J). For example, **if** correct then the order of labelling of nodes B, G and E are 3, 5 and 6 respectively so the working values at F should come from B, G and E in that order. The first working value at F should be their 30 (the Final value at B) + 58 (the weight of the arc BF), the second working value at F should be their 39 (the Final value at G) + 43 (the weight of the arc GF), the third working value at F should be their 50 (the Final value at E) + 30 (the weight of the arc EF). Repeat the process for J (which will have working values from H, G and F with the order of these nodes determined by the candidate's order of labelling at H, G and F).

A1: cao for shortest route (A B D C D G E F J) – must be from A to J (and not stated as a route from J to A)

A1ft: Follow through their final value at A + their final value at J only (so do not award this mark for 156 if it doesn't follow from these two final values)

(c)

M1: Prim (not Kruskal) – First three arcs (CD, BD, DG) correctly chosen, or first four nodes (C, D, B, G) correctly chosen in order. **If any explicit rejections seen at any point then M1 (max) only.** A list of weights alone scores **M0**.

A1: First six arcs correctly chosen in order (CD, BD, DG, EG, GH, EF), **or** all nodes correctly chosen in order (C, D, B, G, E, H, F, J, A)

A1: cso (correct solution only) – all arcs correctly stated (not just nodes) and chosen in the correct order (with no additional, incorrect or repeated arcs)

Misread in (c): Starting at a node other than C scores M1 only – must have the first three arcs (or four nodes) correct (and in the correct order). Therefore applying Prim, starting at A, would give AB, BD, DG,... for **M1** only

(d)

B1: cao (155) (no units required) – **must** follow from the correct arcs stated in (c)

Special Case in (b) – starting at node A:

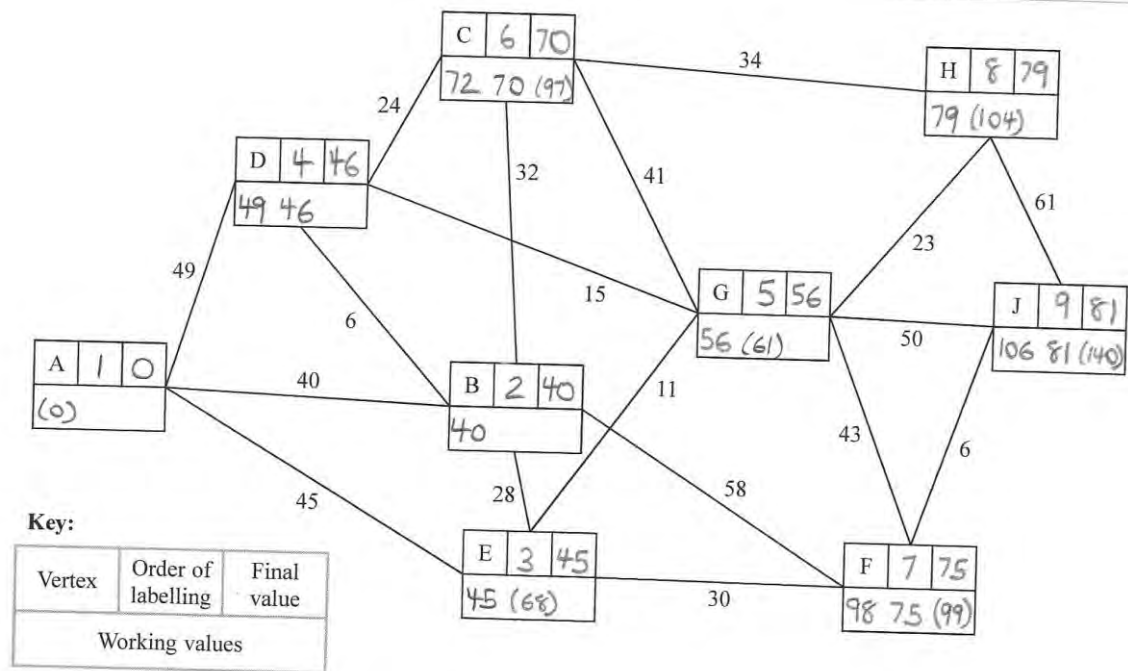
For those candidates starting Dijkstra at A (rather than C) the following marks can be awarded in (b):

M1: A larger working value being replaced by a smaller working value at any **two** of C, D, F, J

A1: All values correct at A, B, E, D and G

A1: All values correct at C, F, H and J

Followed by **A0**, **A0** and **A0** (so max. 3 out of 6 in (b))



| Question | Scheme | Marks | AOs |
|-------------|---|--|--|
| 4(a) | $x \leq 8, \quad 3x + 8y \geq 32, \quad 3y \leq 4x + 5, \quad 4x + 15y \leq 120$ | B2, 1, 0 | 3.3 1.1b |
| | | (2) | |
| 4(b) | <p>Attempt to solve correct two equations to find either optimal vertex</p> <p>Coordinates of 'minimum' point is $\left(\frac{56}{41}, \frac{143}{41}\right)$</p> <p>Coordinates of 'maximum' point is $\left(8, \frac{88}{15}\right)$</p> <p>Setting up a pair of simultaneous equations using their two points and an objective function of the form $ax + by$</p> <p>(If correct $56a + 143b = 883$ oe) $15a + 11b = 100$</p> <p>Objective function is $(P =) 3x + 5y$</p> | <p>M1</p> <p>A1</p> <p>A1</p> <p>dM1</p> <p>A1</p> | <p>3.4</p> <p>1.1b</p> <p>1.1b</p> <p>3.1a</p> <p>2.2a</p> |
| | | (5) | |

(7 marks)**Notes for Question 4****(a)****B1:** Any two correct inequalities. Condone strict inequalities**B1:** All four correct inequalities (not strict)**(b)****M1:** Considering either of the following pairs of simultaneous equations: $3y = 4x + 5, 3x + 8y = 32$ **or** $x = 8, 4x + 15y = 120$

Must find at least one pair of coordinates from either of these two pairs (condone poor algebra in the solving of these equations)

A1: cao $\left(\frac{56}{41}, \frac{143}{41}\right)$ - must be seen exact at some point. They do not have to associate this with being the 'minimum'**A1:** cao $\left(8, \frac{88}{15}\right)$ - must be seen exact at some point. They do not have to associate this with being the 'maximum'**dM1:** Setting up a pair of linear simultaneous equations using their two points. For this mark they must be using their solution of $3y = 4x + 5, 3x + 8y = 32$ together with $\frac{883}{41}$ and their solution of $x = 8, 4x + 15y = 120$ together with $\frac{160}{3}$. Allow use of any two different variables for their pair of linear simultaneous equations. Look out for $\frac{56}{41}x + \frac{143}{41}y = \frac{883}{41}$ and $8x + \frac{88}{15}y = \frac{160}{3}$ which implies the first four marks**A1:** cao – allow just the expression $3x + 5y$ but not any multiple or factor of this (but isw if correct expression is seen first). Allow equal to any other letter but not equal to a value, for example, $3x + 5y = 0$ is **A0**

| Question | Scheme | Marks | AOs |
|--------------|--|------------|------|
| 5.(a) | 1 | B1 | 1.2 |
| | | (1) | |
| (b) | The route is not an example of a path as vertex C appears twice | B1 | 2.4 |
| | | (1) | |
| (c) | As the route contains two roads that need to be traversed twice this means that either the pairing AB, AC or BD, CD needs to be repeated | B1 | 2.1 |
| | $AB + AC = 3x + 6$ and $BD + CD = 3x + 2$ and as $3x + 6 > 3x + 2$ (for all values of x) this means that BD + CD are repeated | B1 | 2.2a |
| | Because two roads are repeated in the shortest inspection route this means that $5x - 8 > 3x + 2$ | M1 | 3.1b |
| | $x > 5$ | A1 | 1.1b |
| | $(20x + 3) + (3x + 2) \leq 189$ | M1 | 3.4 |
| | $x \leq 8$ | A1 | 2.2a |
| | | (6) | |

(8 marks)**Notes for Question 5****(a) B1:** cao

(b) B1: No + correct reason – no bod – must refer to C appearing twice (**not** just that a vertex is repeated) or that it contains the cycle C – F – E – C (**not** just that it contains a cycle). **All technical language must be correct if used for this mark and do not isw any incorrect reasoning** (for example if they imply that a path must pass through every vertex)

(c) B1: Recognising that **one** of the two pairings between B and C containing two arcs will need to be repeated. For example, might state BAC **or** BDC **or** BA, AC **or** BD, DC (as an indication of considering the two odd nodes B and C together with one of the two paths via A and D) **or one** of the expressions $3x + 6$ **or** $3x + 2$ (or correct but unsimplified) seen would score this mark. **Condoned for this mark those candidates who consider the direct arc BC (provided at least one of the pairings via A or D is considered too)**

B1: Correct deduction that BD + CD needs to be repeated (or that AB + AC is not repeated). Allow stating that $3x + 2$ is 'better' than $3x + 6$ **or** simply stating both expressions and selecting $3x + 2$ (but we **must** see both simplified expressions for this mark). This selection of $3x + 2$ (after seeing both expressions) could be implied by forming an equation/inequality **with only this expression**. This mark **cannot** be awarded if either of the other two inequalities/equations e.g. $(20x + 3) + (3x + 6) \leq 189$ **or** $(20x + 3) + (5x - 8) \leq 189$ are formed, unless they are explicitly rejected with the correct reason (that is because $3x + 6 > 3x + 2$ **and** because the route contains two roads). Obtaining $x \leq 7.826...$ and/or $x \leq 7.76$ and simply rejecting these without the valid reasons as stated above does not score this mark

M1: Considers explicitly the direct route between B and C ($5x - 8$) and compares this (in the form of a linear equation or inequality) with either of the two pairings AB + AC **or** BD + CD

A1: cao ($x > 5$)

M1: $(20x + 3) +$ (either their $3x + 6$ or their $3x + 2$) together with 189 (allow equals or any inequality)

A1: cao ($x \leq 8$)

If full marks would have been awarded in (c) but any other inequalities apart from $x > 5$ and $x \leq 8$ are found, then withhold the second B mark

